Universality in strongly correlated quantum systems

Yusuke Nishida (LANL)

Seminar @ University of Chicago

December 10 (2012)
Plan of this talk

1. Universality in physics

2. What is the Efimov effect?
   Keywords: universality, discrete scale invariance, RG limit cycle

3. Efimov effect in quantum magnets

4. Few-body $\Rightarrow$ many-body physics
   New type of crossover physics in three-component Fermi gases
Introduction

1. Universality in physics
2. What is the Efimov effect?
3. Efimov effect in quantum magnets
4. Few-body $\Rightarrow$ many-body physics
(ultimate) Goal of research

Understand physics of few and many particles governed by quantum mechanics
When physics is universal?

A1. Continuous phase transitions $\Leftrightarrow \xi/r_0 \to \infty$

E.g. Water vs. Magnet

Water and magnet have the same exponent $\beta \approx 0.325$

\[
\rho_{\text{liq}} - \rho_{\text{gas}} \sim (T_c - T)^\beta \quad \quad M_\uparrow - M_\downarrow \sim (T_c - T)^\beta
\]
When physics is universal?

A2. Scattering resonances $\leftrightarrow \frac{a}{r_0} \rightarrow \infty$

![Diagram showing the relationship between scattering length, potential depth, and the scattering length-to-ratio $(a/r_0)$](image)
When physics is universal?

A2. Scattering resonances \( \Leftrightarrow \) \( a/r_0 \rightarrow \infty \)

E.g. \(^4\)He atoms vs. proton/neutron

van der Waals force:
\[
a \approx 1 \times 10^{-8} \text{ m } \approx 20 \text{ } r_0
\]

\[
E_{\text{binding}} \approx 1.3 \times 10^{-3} \text{ K}
\]

nuclear force:
\[
a \approx 5 \times 10^{-15} \text{ m } \approx 4 \text{ } r_0
\]

\[
E_{\text{binding}} \approx 2.6 \times 10^{10} \text{ K}
\]

Atoms and nucleons have the same form of binding energy

\[
E_{\text{binding}} \rightarrow - \frac{\hbar^2}{m \ a^2} \quad \text{as} \quad (a/r_0 \rightarrow \infty)
\]

Physics only depends on the scattering length “a”
Efimov effect

1. Universality in physics
2. What is the Efimov effect?
3. Efimov effect in quantum magnets
4. Few-body $\Rightarrow$ many-body physics
Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

Efimov (1970)
Efimov effect

When 2 bosons interact with infinite “a”, 3 bosons always form a series of bound states.
Efimov effect

When 2 bosons interact with infinite “a”, 3 bosons \textit{always} form a series of bound states

Discrete scaling symmetry
Renormalization group limit cycle

Renormalization group flow diagram in coupling space

![Diagram showing renormalization group fixed point and limit cycle.]

**RG fixed point**
⇒ Scale invariance
E.g. critical phenomena

**RG limit cycle**
⇒ Discrete scale invariance
E.g. Efimov effect

Rare manifestation in physics!
Where Efimov effect appears?

× Originally, Efimov considered
  $^3\text{H}$ nucleus ($\approx 3\, \text{n}$) and $^{12}\text{C}$ nucleus ($\approx 3\, \alpha$)

△ $^4\text{He}$ atoms ($a \approx 1 \times 10^{-8}\, \text{m} \approx 20\, r_0$) ?

2 trimer states were predicted
1 was observed (1994)

$E_b = 125.8\, \text{mK}$

$E_b = 2.28\, \text{mK}$

Ultracold atoms!
Ultracold atom experiments

Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will.
Ultracold atom experiments

Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will.

✓ Interaction strength by Feshbach resonances

![Graph showing scattering length vs. magnetic field (B) with data points and curves indicating the universal regime.](image)

C.A. Regal & D.S. Jin
PRL90 (2003)
Ultracold atom experiments

Florence group for $^{39}$K (2009)

Bar-Ilan University for $^7$Li (2009)

Rice University for $^7$Li (2009)

Discrete scaling & Universality!
Efimov effect is “universal”?

- Efimov effect is “universal”
  = appears regardless of microscopic details
  (physics technical term)

- Efimov effect is not “universal”
  universal = present or occurring everywhere
  (Merriam-Webster Online)

Can we find the Efimov effect in other physical systems?
Efimov effect in quantum magnets

1. Universality in physics
2. What is the Efimov effect?
3. Efimov effect in quantum magnets
4. Few-body $\Rightarrow$ many-body physics
Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

\[ H = - \sum_r \left[ \sum_{\hat{e}} (J_S^+ S^-_{r+\hat{e}} + J_z S^z_r S^z_{r+\hat{e}}) + D(S^z_r)^2 - B S^z_r \right] \]

- **Exchange anisotropy**
- **Single-ion anisotropy**

**Spin-boson correspondence**

- Fully polarized state \( (B \rightarrow \infty) \)
  \( \Leftrightarrow \) No boson = vacuum

- \( N \) spin-flips
  \( \Leftrightarrow \) \( N \) bosons = magnons
Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

\[ H = - \sum_r \left[ \sum_{\hat{e}} (J S^+_r S^-_{r+\hat{e}} + J_z S^z_r S^z_{r+\hat{e}}) + D (S^z_r)^2 - B S^z_r \right] \]

- xy-exchange coupling \(\Leftrightarrow\) hopping
- z-exchange coupling \(\Leftrightarrow\) neighbor attraction
- single-ion anisotropy \(\Leftrightarrow\) on-site attraction

\( N \) spin-flips \(\Leftrightarrow\) \( N \) bosons = magnons
Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

\[ H = - \sum_r \left[ \sum_{\hat{e}} \left( J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z \right) + D (S_r^z)^2 - B S_r^z \right] \]

xy-exchange coupling \iff hopping

z-exchange coupling \iff neighbor attraction

single-ion anisotropy \iff on-site attraction

Tune these couplings (with pressure) to induce scattering resonance between two magnons

\[ \Rightarrow \text{Three magnons show the Efimov effect} \]
Two-magnon resonance

Scattering length between two magnons

\[
\frac{a_s}{a} = \frac{\frac{3}{2\pi} \left[ 1 - \frac{D}{3J} - \frac{J_z}{J} \left( 1 - \frac{D}{6SJ} \right) \right]}{2S - 1 + \frac{J_z}{J} \left( 1 - \frac{D}{6SJ} \right) + 1.52 \left[ 1 - \frac{D}{3J} - \frac{J_z}{J} \left( 1 - \frac{D}{6SJ} \right) \right]}
\]

Two-magnon resonance \((a_s \to \infty)\)

- \(J_z/J = 2.94\) (spin-1/2)
- \(J_z/J = 4.87\) (spin-1, \(D=0\))
- \(D/J = 4.77\) (spin-1, ferro \(J_z=J>0\))
- \(D/J = 5.13\) (spin-1, antiferro \(J_z=J<0\))
- ...
Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies $E_n$

- Spin-1/2
  
  "$J_z/J = 2.94$"

- Spin-1, $D=0$
  
  "$J_z/J = 4.87$"

- Spin-1, $J_z=J>0$
  
  "$D/J = 4.77$"

- Spin-1, $J_z=J<0$
  
  "$D/J = 5.13$"
Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies $E_n$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$E_n/J$</th>
<th>$\sqrt{E_{n-1}/E_n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-2.09 \times 10^{-1}$</td>
<td>—</td>
</tr>
<tr>
<td>1</td>
<td>$-4.15 \times 10^{-4}$</td>
<td>22.4</td>
</tr>
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- **Spin-1/2**

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- **Spin-1, $D=0$**

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<tr>
<td>0</td>
<td>$-5.50 \times 10^{-2}$</td>
<td>—</td>
</tr>
<tr>
<td>1</td>
<td>$-1.16 \times 10^{-4}$</td>
<td>21.8</td>
</tr>
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- **Spin-1, $J_z=J>0$**

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<td>$-4.36 \times 10^{-3}$</td>
<td>—</td>
</tr>
<tr>
<td>1</td>
<td>$-8.88 \times 10^{-6}$</td>
<td>22.2</td>
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- **Spin-1, $J_z=J<0$**
Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies $E_n$

- **Spin-1/2**

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- **Spin-1, D=0**

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Universal scaling law by ~ 22.7 confirms they are **Efimov states**!

Observable with usual spectroscopic measurements
Short summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

-ultracold atoms
-nuclear/particle
-condensed matter

Atomic BEC (1995 🇺🇸)
Efimov effect (2006 🇺🇸)

Magnon BEC (1999 🇯🇵)
Efimov effect (201? 🇯🇵)
Short summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

ultracold atoms

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Atomic BEC (1995 🇺🇸)

Efimov effect (2006 🇺🇸)

Magnon BEC (1999 🇯🇵)

Efimov effect (201? ?)

New link between atomic and magnetic systems
Short summary

How do they interplay?

Many-body physics
- magnetism
- superfluidity
- superconductivity
- ...

Few-body physics
New crossover physics in 3-component FGs

1. Universality in physics
2. What is the Efimov effect?
3. Efimov effect in quantum magnets
4. Few-body $\Rightarrow$ many-body physics
BCS-BEC crossover

• 2-component Fermi gas
  loosely bound Cooper pairs  tightly bound dimers

Jin Group at JILA
BCS-BEC crossover

- 3-component Fermi gas

- loosely bound Cooper pairs

- tightly bound dimers

- unpaired atoms
BCS-BEC crossover

• 3-component Fermi gas

loosely bound Cooper pairs

unpaired atoms

“Atom-trimer continuity” = New crossover physics!
3-component Fermi gas

- 3 spin states (i=1,2,3) of $^6$Li atoms near a Feshbach resonance:

$$f(k) = \frac{-1}{ik + \frac{1}{a}}$$

- $a_{12} = a_{23} = a_{31}$

3-component Fermi gas

- 3 spin states \((i=1, 2, 3)\) of \(^6\text{Li}\) atoms near a Feshbach resonance:

\[
f(k) = \frac{-1}{ik + \frac{1}{a}}
\]

- \(a_{12} = a_{23} = a_{31} \Rightarrow \text{SU}(3) \times \text{U}(1)\) invariance

\[
\mathcal{L} = \psi_i^\dagger \left( i\partial_t + \frac{\nabla^2}{2m} \right) \psi_i + \frac{2\pi a}{m} \psi_i^\dagger \psi_j^\dagger \psi_j \psi_i
\]

- **Problem!** 3 fermions form an infinitely deep bound state (Thomas collapse)

No many-body ground state :-(

- [Image of 3-component Fermi gas](image-url)
3-component Fermi gas

• 3 spin states (i=1,2,3) of $^6\text{Li}$ atoms near a “narrow” Feshbach resonance:

$$f(k) = \frac{-1}{ik + \frac{1}{a}}$$

$$f(k) = \frac{-1}{ik + \frac{1}{a} + Rk^2}$$

$$r_{\text{eff}} = -2R$$ is the effective range

• $R$ regularizes short-distance behaviors
  ($\Rightarrow$ no Thomas collapse)

Universal many-body ground state
(depends only on $a$, $R$, $k_F$)
Phase diagram

- weak attraction
- strong attraction
- broad resonance
- narrow resonance
Phase diagram

- BCS superfluid
  - weak attraction
  + Fermi surface

- BEC superfluid
  - strong attraction
  + no Fermi surface

Trimer Fermi gas

$Rk_F$

$n/a k_F$

↑ narrow resonance
Phase diagram

- Large $R k_F$ expansion
- BCS superfluid + Fermi surface
- Trimer Fermi gas
- BEC superfluid + no Fermi surface
Phase diagram

\[
\frac{1}{ak_F} = \frac{4}{\Gamma(1/4)^2} \left( \frac{4\pi}{Rk_F} \right)^{1/4}
\]

- BCS superfluid + Fermi surface
- BEC superfluid + no Fermi surface
- Trimer Fermi gas
Phase diagram

- BCS superfluid
- BEC superfluid
- no Fermi surface

Trimer Fermi gas

Dilute limit $k_F \to 0$

$Rk_F$

$1/ak_F$

$mR^2 \frac{E_N}{N}$

$0$

$R_*$

$\frac{1}{a}$

$N=2$

$N=3$
Phase diagram

\[ \frac{1}{ak_F} = -\frac{0.0917}{Rk_F} \]
Phase diagram

- BCS superfluid
- Fermi surface
- BEC superfluid
- no Fermi surface

Trimer Fermi gas

dimer SF + trimer FS

$R k_F$

$1/ak_F$

$E_N$ $mR^2/N$

$R^*_a$

$N=3$ $N=2$
Phase diagram

- BCS superfluid + Fermi surface
- BEC superfluid + no Fermi surface
- Trimer Fermi gas
- Dimer SF + trimer FS
Unpaired fermions are \textit{atom}-like \((m_{\text{eff}} \sim m)\)

Unpaired fermions are \textit{trimer}-like \((m_{\text{eff}} \sim 3m)\)

"Atom-trimer continuity" = New crossover physics!
Continuity of Quark and Hadron Matter

Thomas Schäfer and Frank Wilczek

School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540

(Received 30 November 1998)

We review, clarify, and extend the notion of color-flavor locking. We present evidence that for three degenerate flavors the qualitative features of the color-flavor locked state, reliably predicted for high density, match the expected features of hadronic matter at low density. This provides, in particular, a controlled, weak-coupling realization of confinement and chiral symmetry breaking in this (slight) idealization of QCD. [S0031-9007(99)09191-7]

PACS numbers: 12.38.Aw

In a recent study [1] of QCD with three degenerate flavors at high density, a new form of ordering was predicted, wherein the color and flavor degrees of freedom become rigidly correlated in the ground state: color-flavor locking. This prediction is based on a weak-coupling analysis using a four-fermion interaction with quantum numbers abstracted from one gluon exchange. One expects that such a weak-coupling analysis is appropriate at high density, for the following reason [2,3]. Tentatively assuming that the quarks start out in a state close to their free quark state, i.e., with large Fermi surfaces, one finds that the relevant interactions, which are scattering the states near the Fermi surface, for the most part involve large momentum transverse quantum numbers, including integral electric charge. Thus, the gluons match the octet of vector mesons, the quark octet matches the baryon octet, and an octet of collective modes associated with chiral symmetry breaking matches the pseudoscalar octet. However, there are also a few apparent discrepancies: there is an extra massless singlet scalar, associated with the spontaneous breaking of baryon number (superfluidity); there are eight rather than nine vector mesons (no singlet); and there are nine rather than eight baryons (extra singlet). We will argue that these “discrepancies” are superficial — or rather that they are features, not bugs.

Let us first briefly recall the fundamental concepts of continuity [4], even weak couplings near the Fermi surface can be very strong in nuclei. So, for the remainder of this paper we shall concentrate on it. The primary condensate takes the form [1] of a quark singlet, a flavor singlet, a color singlet, a flavor octet, a color octet, etc.) whose nonzero values emerge from a wave function [1] description of the energy, i.e., with Hamiltonian [1] acting on a wave function [1] with appropriate boundary conditions. The wave function [1] is the vacuum expectation value of a quark operator, is real, is spherically symmetric, and is of form [1] of a wave function [1] with appropriate boundary conditions.

New link between atomic and nuclear systems !
Summary of this talk

Keywords: strong correlation & universality

- condensed matter
- ultracold atoms
- nuclear/particle

✓ Efimov effect in quantum magnets
  New link between atomic and magnetic systems

✓ Atom-trimer continuity in 3-component FGs
  New link between atomic and nuclear systems

New ideas wanted!