“Hard probes” of strongly-interacting atomic gases

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This talk is about ... 

Energetic atom propagating through strongly-interacting atomic gases

✓ energetic atom ⇒ many-body system
✓ energetic atom ⇐ many-body system
This talk is about ...

Energetic atom propagating through strongly-interacting atomic gases

1. Motivations
   ✓ close connection to nuclear physics
   ✓ useful to locally probe many-body physics

2. Ideas & methods
   ✓ large-momentum expansion à la OPE

3. Results and discussions
   ✓ energetic atom $\Rightarrow$ many-body system
   ✓ energetic atom $\Leftarrow$ many-body system
Motivations
How to probe target systems?

Typically in ultracold atoms, light is used to probe atoms.
How to probe target systems?

Often in nuclear and particle physics, “hard probes” (= high-energy particles) are used to probe target systems.

How to probe target systems?

“Hard probes” (= enegetic atoms) may be useful to probe target atomic gases

What can be probed? And how?

• Microscopic Hamiltonian is **simple**

\[
H = - \int dx \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \frac{\nabla^2}{2m} \psi_\sigma + g \int dx dy \psi_\uparrow^\dagger \psi_\uparrow(x) \delta(x - y) \psi_\downarrow^\dagger \psi_\downarrow(y)
\]

• **Systematic** approach is possible to develop strict results (without relying on phenomenology)

• Seen as an **idealization** of nuclear physics (by neglecting relativistic effects, isospins, finite range of interactions, 3-body forces, ...)

Energetic atom $\Rightarrow$ many-body system
Probe atomic gas with atoms

Shoot a probe atom into the target atomic gas and measure its differential scattering rate.

What can we learn from the scattering data on the (strongly-interacting) target atomic gas?
Probe atomic gas with atoms

Shoot a probe atom into the target atomic gas and measure its differential scattering rate

Large $k \gg n^{1/3} \Rightarrow$ Few-body scattering problems

$$\frac{d\Gamma(k)}{d\Omega} = \cdots$$
Leading contribution

Shoot a probe atom into the target atomic gas and measure its differential scattering rate

Large $k \gg n^{1/3} \Rightarrow$ Few-body scattering problems

$$\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + \cdots$$
Sub-leading contribution

Shoot a probe \textbf{atom} into the target atomic gas and measure its differential scattering rate

\[ \frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta) \frac{C}{k^2} + \cdots \]

Large \( k \gg n^{1/3} \) \( \Rightarrow \) Few-body scattering problems
What is “C”?

Probability of finding 2 particles at small separation

- noninteracting gas: $\langle \hat{n}(r)\hat{n}(0) \rangle = n^2$

- interacting gas: $\langle \hat{n}(r)\hat{n}(0) \rangle \rightarrow \frac{C}{(4\pi|r|)^2}$

Anomally enhanced probability is quantified by the “contact density” $C$

Important characteristic of strongly-int atomic gases

**Viewpoint: How the tail wags the dog in ultracold atomic gases**

**Eric Braaten**, Department of Physics, Ohio State University, Columbus, OH 43210 USA and and Bethe Center for Theoretical Physics, University of Bonn, Bonn, Germany

Published February 2, 2009 | Physics 2, 9 (2009) | DOI: 10.1103/Physics.2.9

Recent calculations of the properties of ultracold atoms have revealed how two-body interactions at very short distances determine essential properties of many-body systems.

The development of the field of ultracold atoms has opened up new frontiers in both few-body and many-body physics. Of particular interest are the various ways that the physics of few-body systems can influence and be influenced by many-body systems.

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**Universal properties of the ultracold Fermi gas**

Shizhong Zhang and Anthony J. Leggett


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**Viewpoint: Fermi gases as a test bed for strongly interacting systems**

**Daniel E. Sheehy**, Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA

Published June 7, 2010 | Physics 3, 48 (2010) | DOI: 10.1103/Physics.3.48

A new perspective on strongly interacting fermions emerges from the experimental confirmation of a universal formula.

Some of the most vexing present-day problems in physics center on understanding the many-body properties and phases of strongly interacting fermions. Part of the difficulty arises from the fact that while the interactions are relatively strong, the number of particles is small enough so that quantum fluctuations play an important role.

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**Verification of Universal Relations in a Strongly Interacting Fermi Gas**


Formulations à la OPE

- **scattering rate**: $\Gamma(k) = -2 \text{Im} \Sigma(k)$

- **optical theorem**: $\Gamma(k) = \int d\Omega \frac{d\Gamma(k)}{d\Omega}$

$$iG(k) = \int dx \ e^{ikx} \langle T \psi(x)\psi^\dagger(0) \rangle$$

$$= \sum_i A_i(k) \langle O_i \rangle$$

$$n = \langle \psi^\dagger \psi \rangle, \ C = \langle (\psi^\dagger \psi)^2 \rangle, \ ...$$

Lowest few $O_i$ are needed at large $k$

**Systematic large-k expansion !**
Differential scattering rate

$$\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta) \frac{C}{k^2} + \cdots$$

Many-body physics

Few-body physics

Few-body physics plays an important role to probe many-body physics!
Differential scattering rate

\[
\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta, k/\kappa_*) \frac{C}{k^2} + \cdots
\]

For zero-range interactions

\( f(\theta) \)

\( g(\theta, 1.8) \)

\( \cos \theta \)

forward scattering
(\( \theta < 90^\circ \)) only

backward scattering
(\( \theta > 90^\circ \)) possible

Efimov effect
Differential scattering rate

\[ \frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta, k/k_*) \frac{C}{k^2} + \cdots \]

For zero-range interactions

Efimov effect

\[ f(\theta) \]

\[ g(\theta, 1.8) \]

Backward scattering rate measures contact density

New local probe of strongly-int atomic gases
Differential scattering rate

\[
\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta, k/\kappa_*) \frac{C}{k^2} + 16 \Theta(\cos \theta) \left( 2 \cos \theta \hat{k} + \hat{p} \right) \cdot \frac{\vec{j}}{k^2} + \cdots
\]

Azimuthal ($\varphi$) anisotropy reveals currents in many-body states
Ultracold atom “colliders”

Duke (2011)

MIT (2011)

NIST (2012)

Otago (2012)
Ultracold atom “colliders”

“A laser based accelerator for ultracold atoms”

University of Otago
(New Zeeland)
Optics Letters (2012)
Energetic atom \Rightarrow many-body system
Energetic atom $\leftrightarrow$ many-body system
Energy in a Fermi gas

\[ E_{\uparrow}(k) = \frac{k^2}{2m} \]

\[ \Gamma_{\uparrow}(k) = 0 \]

Energetic spin-\(\uparrow\) fermion
Energy in a Fermi gas

\[ E_{\uparrow}(k) = \frac{k^2}{2m} \]

\[ \Gamma_{\uparrow}(k) = 0 \]

Energetic spin-\(\uparrow\) fermion in a Fermi gas with scattering length \(a\)
Energy in a Fermi gas

\[ E_{\uparrow}(k) = \left[ 1 + 32\pi \frac{n_{\downarrow}}{ak^4} - 7.54 \frac{C}{k^4} + O(k^{-6}) \right] \frac{k^2}{2m} \]

\[ \Gamma_{\uparrow}(k) = \left[ 32\pi \left( 1 - \frac{4}{a^2k^2} \right) \frac{n_{\downarrow}}{k^3} + 44.2 \frac{C}{ak^5} + O(k^{-6}) \right] \frac{k^2}{2m} \]

Energetic spin-\(\uparrow\) fermion
in a Fermi gas with scattering length \(a\)
How large is large?

\[ E_\uparrow(k) = \left[ 1 + 32\pi \frac{n_\downarrow}{ak^4} - 7.54 \frac{C}{k^4} + O(k^{-6}) \right] \frac{k^2}{2m} \]

Comparison of \( E(k)/\epsilon_F \) with QMC  

P. Magierski et al., PRL (2011)

\( (a_f k_F)^{-1} = 0 \quad T/\epsilon_F = 0.15 \)
\( C/k_F^4 = 0.110 \)  
Goulko et al, Lattice2010

\( (a_f k_F)^{-1} = 0.2 \quad T/\epsilon_F = 0.19 \)
\( C/k_F^4 = 0.156 \)  
Gandolfi et al, PRA (2011)

Reasonable agreement even at \( k/k_F > 1.5 \)!
Summary of this talk

- Energetic atoms ⇒ New tool to locally probe strongly-interacting atomic gases
- Systematic large-k expansions are possible
  ✓ backward scattering ⇒ contact density
  ✓ azimuthal anisotropy ⇒ current density
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“Hard probes” are useful to reveal short-range pair correlations both in nuclei and atomic gases

Summary of this talk

Nuclear physics

Ultracold atoms

New ideas wanted!